A Combined Global and Local Identification Approach for LPV Systems ⋆

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Abstract: This paper tackles the problem of identifying linear parameter-varying (LPV) systems by combining data originating from global and local identification experiments into a nonlinear least-squares problem. One extreme of the approach results in a model optimal with respect to the system behavior under varying scheduling parameter conditions, while the other gives a model being a good approximation of system behavior for fixed scheduling parameter. When measurements from global and local experiments are available, a compromise between the two objectives is achieved. Numerical and experimental validations, accompanied by comparisons with existing LPV identification methods show the potential of the developed approach.

Keywords: Nonlinear systems, state-space models, identification algorithms, optimization problems, parameter estimation, time-domain responses, frequency responses, subspace methods, validation.

1. INTRODUCTION

Linear parameter-varying (LPV) systems are nonlinear systems described by a linear model coefficients of which vary as a function of the so called *scheduling parameters*. These time-varying parameters determine the system's operating point. The linear property makes LPV systems attractive for modern industrial control with applications in aircrafts, automotive engines, robotics, wind turbines and similar, since they inherit some features of the well studied linear time-invariant (LTI) systems.

The literature on LPV system identification typically distinguishes between two different identification approaches - global and local. The global techniques (see e.g. Bamieh and Giarre (2002), Felici et al. (2007), Sznaier et al. (2000), Verdult and Verhaegen (2002)) directly identify an LPV model based on data obtained from an experiment where both input and scheduling parameters are continuously changing; we call these data *global*. The local identification techniques (see e.g. de Caigny et al. (2013), Lovera and Mercère (2007), Steinbuch et al. (2003)) typically consist of two steps. In the first step, several LTI models are identified based on *local* input-output data obtained for various fixed values of the scheduling. In the second step, these LTI models are interpolated yielding a parameter varying model. The existing methods usually consider different identification settings, use different model structures and target different objectives, what hinders a fair comparison of the performance.

Both approaches have their advantages and disadvantages. The global approach may offer high accuracy in predicting the system behavior under changing scheduling parameter conditions, in exhange for involved experiment design in terms of ensuring persistency of the excitation. What goes in its favour is also the fact that the *dynamic scheduling dependency*, that is, system's dependency on time-shifted instances of the scheduling parameters, can only be detected through a global identification experiment. The local approach, on the other hand, can accurately identify only systems with *static scheduling dependency* - dependency on the instantaneous time values of the scheduling parameters, but can to a large extent rely on the well-studied linear time-invariant (LTI) identification methods.

Although different, data originating from global and local experiments both provide valuable information that, when put together, gives a more complete picture of the system at hand. The main contribution of this paper is exploring the possibility of combining the two approaches, together with drawing attention to the capacity of the nonlinear least-squares (NLS) identification framework for LPV systems (see Verdult and Verhaegen (2001), Verdult (2002), and Verdult et al. (2003)). We chose the concerned framework for several reasons: it easily combines data originating from different experiments, the data it engages can be in the time and/or the frequency domain, it allows to emphasize particular experiments by simply employing weighting matrices, and the solution can be found efficiently using the well-known Levenberg-Marquardt algorithm. In this way, it is possible to balance between the importance of the system's behavior under changing scheduling parameter conditions, and the behavior for fixed operating conditions. The model we propose belongs to the output error model family, proven to be suitable for nonlinear least-squares optimization (see Verdult (2002)).

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The paper is organized as follows. Section 2 describes the chosen LPV output error model structure, covering static and dynamic scheduling dependency. Section 3 introduces the optimization problem that combines data originating from global and local identification experiments, and suggests how to efficiently find the solution. Section 4 covers two numerical examples. In the first example only measurements taken from a global experiment are used, while the second example regards measurements taken from local experiments. In both cases, the performance of the obtained model is compared with the performance of the model resulting from a state-of-the art method for the specific type of identification data used. Finally, in Section 5 a real LPV system is identified by combining measurements taken from global and local experiments. The obtained results form the bottom line for the conclusions conveyed in Section 6.

2. LPV OUTPUT ERROR MODEL STRUCTURE

In this paper we focus on the following fully parameterized discrete time LPV model:

$$\begin{cases} x(t+1) = (\mathscr{A} \diamond p)(t) \cdot x(t) + (\mathscr{B} \diamond p)(t) \cdot u(t) \\ y(t) = (\mathscr{C} \diamond p)(t) \cdot x(t) + (\mathscr{D} \diamond p)(t) \cdot u(t) + e(t), \end{cases} \tag{1}$$

where $x(t) \in \mathbb{R}^n$, $u(t) \in \mathbb{R}^r$, $y(t) \in \mathbb{R}^l$, $p(t) \in \mathbb{R}^{N_p}$, $e(t) \in \mathbb{R}^l$, are the state vector, the input and output vectors, the scheduling parameter vector and the zero mean white measurement noise, at time instance t.

The state-space matrices of the introduced model are parameter-varying:

$$(\mathscr{A} \diamond p)(t) = A^{(0)} + \sum_{i=1}^{N_p} \sum_{j=1}^{N_b} A^{(i,j)} f_j(p_i(t), p_i(t-1), ..., p_i(t-n_d)),$$

$$(\mathscr{B} \diamond p)(t) = B^{(0)} + \sum_{i=1}^{N_p} \sum_{j=1}^{N_b} B^{(i,j)} f_j(p_i(t), p_i(t-1), ..., p_i(t-n_d)),$$

$$(\mathscr{C} \diamond p)(t) = C^{(0)} + \sum_{i=1}^{N_p} \sum_{j=1}^{N_b} C^{(i,j)} f_j(p_i(t), p_i(t-1), ..., p_i(t-n_d)),$$

$$(\mathscr{D} \diamond p)(t) = D^{(0)} + \sum_{i=1}^{N_p} \sum_{j=0}^{N_b} D^{(i,j)} f_j(p_i(t), p_i(t-1), ..., p_i(t-n_d)),$$

where $A^{(0)} \in \mathbb{R}^{n\times n}$, $A^{(i,j)} \in \mathbb{R}^{n\times n}$, $B^{(0)} \in \mathbb{R}^{n\times r}$, $B^{(i,j)} \in \mathbb{R}^{n\times r}$, $C^{(0)} \in \mathbb{R}^{l\times n}$, $C^{(i,j)} \in \mathbb{R}^{l\times n}$, $D^{(0)} \in \mathbb{R}^{l\times r}$, $D^{(i,j)} \in \mathbb{R}^{l\times r}$; N_b is the number of basis functions employed for parameterization, and n_d is the number of time-shifts of the scheduling parameters.

For simplicity, the above model does not allow a basis function f_j to depend on more than one scheduling parameter, which means that e.g. power product between the elements of the scheduling parameter vector are not encountered. Nevertheless, the extension follows naturally.

3. NONLINEAR LEAST SQUARES PROBLEM FORMULATION

Local identification data can be either time or frequency domain data. Global identification data are mostly used in the time domain, although they can be treated in the frequency domain as well, see Goos et al. (2014). We will only use the time domain for the data coming from global experiments, although this does not exclude possible extensions in the future.

Assume that N_t different sets of time domain data and N_f different sets of frequency domain data are available.

First consider time domain data, which can originate from either local or global experiments. The difference between the response of the LPV model (1) y to the input u in the q_t^{th} experiment, and the measured output y_m, equals:

$$\boldsymbol{\varepsilon}_\mathrm{t}^{q_\mathrm{t}}(\Theta) = \mathbf{y}^{q_\mathrm{t}}(\Theta) - \mathbf{y}_\mathrm{m}^{q_\mathrm{t}}, \tag{2}$$

where

$$\Theta = [vec(A);\ vec(B);\ vec(C);\ vec(D)]. \tag{3}$$

Second assume N_f local experiments providing frequency domain data. If $\mathbf{G}_\mathrm{m}^{\mathbf{q_f}}$ is the value of the system's complex frequency response function (FRF) resulting from the q_f^{th} local experiment, the error of the associated model frequency response $\mathbf{G}^{q_\mathrm{f}}(\Theta)$ then equals:

$$\boldsymbol{\varepsilon}_\mathrm{f}^{q_\mathrm{f}}(\Theta) = \mathbf{G}^{q_\mathrm{f}}(\Theta) - \mathbf{G}_\mathrm{m}^{q_\mathrm{f}}. \tag{4}$$

A weighted least squares criterion that combines global and local experiments, from the time and frequency domain, can now be formulated.

$$V(\Theta) = \frac{1}{2}\left(\sum_{q_\mathrm{t}} (\boldsymbol{\varepsilon}_\mathrm{t}^{q_\mathrm{t}}(\Theta))^\mathrm{T} W_\mathrm{t}^{q_\mathrm{t}} \boldsymbol{\varepsilon}_\mathrm{t}^{q_\mathrm{t}}(\Theta) + \sum_{q_\mathrm{f}} (\boldsymbol{\varepsilon}_\mathrm{f}^{q_\mathrm{f}}(\Theta))^\mathrm{H} W_\mathrm{f}^{q_\mathrm{f}} \boldsymbol{\varepsilon}_\mathrm{f}^{q_\mathrm{f}}(\Theta)\right) \tag{5}$$

W_t is a time domain weighting that allows to emphasize a time span of interest, or a particular experiment. When no specific weighting is required by the user, a constant that normalizes the time domain error is applied:

$$W_\mathrm{t}^{q_\mathrm{t}} = \left(\sum_{q_\mathrm{t}} \|\mathbf{y}_\mathrm{m}^{q_\mathrm{t}}\|^2\right)^{-1}. \tag{6}$$

W_f is a frequency domain weighting that allows to emphasize a frequency range of interest, or a particular local experiment. The recommended weighting is the one that normalizes the error in the frequency domain, that is

$$W_\mathrm{f}^{q_\mathrm{f}} = \left(\sum_{q_\mathrm{f}} \|\mathbf{G}_\mathrm{m}^{q_\mathrm{f}}\|^2\right)^{-1}. \tag{7}$$

The optimal set of parameter estimates Θ^* is obtained by solving the following nonlinear least squares problem

$$\Theta^* = arg\min_{\Theta} V. \tag{8}$$

The solution of such problem is typically obtained using the iterative Levenberg-Marquardt algorithm. Non-uniqueness of the fully parameterized state-space model (1) may sometimes cause numerical problems due to a rank-deficient Jacobian. This can be overcome by identifying the representations that are input-output equivalent to the one given by $\{A, B, C, D\}$ at a certain step, and then considering only their orthogonal complements as potential solutions in the next step. In this way, we avoid representations that lead to unchanged values of the objective function. This functionality is known as the Data Driven Local Coordinates (DDLC) - see Helmersson and McKelvey (1999), McKelvey et al. (2004), or projected gradient search - see Verdult and Verhaegen (2001), Verdult et al. (2003), and is included in our implementation of the Levenberg-Marquardt algorithm.

4. NUMERICAL VALIDATION

The proposed nonlinear least squares identification technique is demonstrated on two discrete time LPV systems: a multiple-input multiple-output (MIMO), and a single-input single-output (SISO) system. In the first example only data taken from a global experiment are used. The goal is to show the improvement that can be achieved with respect to a state-of-the-art global LPV identification technique. The aim of the second example, on the other hand, is to compare the developed method with a state-of-the art local identification technique using identification data from local experiments only. The code of the developed NLS method is implemented in MATLAB v8.0.0.783 (R2012b), partially in C/C++ MEX-files, and executed on the 64-bit Operating System with Intel® Core™ i5-3210M CPU @ 2.50 GHz and 8 GB of RAM.

4.1 MIMO LPV model identification using global data

The device under test is a fourth-order discrete time LPV system with two inputs, three outputs and three scheduling parameters. It has been presented and identified in (Verdult and Verhaegen (2002)) using a global approach with randomly changing scheduling parameters. We keep the same experiment design and first identify the system using the enhanced subspace technique (van Wingerden and Verhaegen (2009)) implemented in (Houtzager et al. (2012)). The obtained subspace LPV model is used as the initial guess for the NLS problem (8) with the weighting (6) we then solve.

The measurements are $N = 1000$ samples long, and both a noise-free and a noisy data set are considered. The output measurement noise in the second data set is a zero mean uncorrelated Gaussian noise, yielding a signal-to-noise ratio of 40 dB. For analysis, the two cases are repeated 100 times, each time using new realizations of the input signals, the scheduling signals and the noise in the second case. In the end, all the identified models are validated on the same noise-free data set generated in a new global experiment. As measure of the quality of the identified models, we use: the mean squared error

$$\mathrm{MSE} := \frac{1}{N}\|\mathbf{y}_\mathrm{m} - \mathbf{y}(\Theta)\|_2^2, \tag{9}$$

with N being the number of data samples, the best fit percentage

$$\mathrm{BFT} := \max\left(1 - \frac{\|\mathbf{y}_\mathrm{m} - \mathbf{y}(\Theta)\|_2}{\|\mathbf{y}_\mathrm{m} - \overline{\mathbf{y}_\mathrm{m}}\|_2}, 0\right) \times 100\%, \tag{10}$$

and the variance accounted for defined as

$$\mathrm{VAF} := \max\left(1 - \frac{\mathrm{var}(\mathbf{y}_\mathrm{m} - \mathbf{y}(\Theta))}{\mathrm{var}(\mathbf{y}_\mathrm{m})}, 0\right) \times 100\%, \tag{11}$$

where $\mathrm{var}(\cdot)$ denotes the variance of a signal.

Table 1 and Table 2 contain the averaged values of the MSE, BFT and VAF calculated using 100 LPV models identified with the enhanced subspace technique (van Wingerden and Verhaegen (2009)) and further improved with the NLS optimization. Table 1 considers the case without the measurement noise, while Table 2 covers the noisy data.

By looking at the values of the MSE in Table 1, one can see that the presented NLS model significanly improves the model accuracy, and this improvement is obtained after reasonable number of iterations (N_iter) and in a reasonable computation time (T_cmp). The values of the BFT indicate the same, while the VAF values do not allow us to make a distinction. A certain amount of accuracy is lost when the measurement noise is added, which is again, most obvious from the MSE. The message, however, stays the same.

Table 1. Average MSE, BFT and VAF of the subspace model (Θ_0) obtained through 100 identification proceedings with SNR = ∞ dB, compared with the values obtained with the NLS optimization (Θ^*)

Parameter set	$\overline{\mathrm{MSE}}$	$\overline{\mathrm{BFT}}$	$\overline{\mathrm{VAF}}$
Θ_0	$7.06 \cdot 10^{-5}$	99.96	100
Θ^*	$5.03 \cdot 10^{-14}$	100	100

$\overline{N_\mathrm{iter}} = 16.28,\ \overline{T_\mathrm{cmp}} = 11.61\mathrm{s}$

Table 2. Average MSE, BFT and VAF of the subspace model (Θ_0) obtained through 100 identification proceedings with SNR = 40 dB, compared with the values obtained with the NLS optimization (Θ^*)

Parameter set	$\overline{\mathrm{MSE}}$	$\overline{\mathrm{BFT}}$	$\overline{\mathrm{VAF}}$
Θ_0	0.16	97.20	99.91
Θ^*	0.01	99.32	99.99

$\overline{N_\mathrm{iter}} = 21.75,\ \overline{T_\mathrm{cmp}} = 15.4\mathrm{s}$

4.2 SISO LPV model identification using local data

The considered SISO LPV system is a fictive discrete time LPV system of second order, the behavior of which is described by (1) with the state-space matrices:

$$\begin{gathered}
A^{(0)} = \begin{bmatrix} -1.1 & 0 \\ 0 & 0.1 \end{bmatrix},\ A^{(1,1)} = \begin{bmatrix} 0.1 & 0 \\ 0 & -0.1 \end{bmatrix}, \\
A^{(1,2)} = \begin{bmatrix} 0.002 & 0 \\ 0 & 0.005 \end{bmatrix},\quad B^{(0)} = \begin{bmatrix} 1.2 \\ 3.3 \end{bmatrix},\ B^{(1,1)} = \begin{bmatrix} 0.02 \\ 0 \end{bmatrix}, \\
B^{(1,2)} = \begin{bmatrix} 0 \\ 0.006 \end{bmatrix},\quad C^{(0)} = [0.4 \quad 1],\ C^{(1,1)} = [0.8 \quad 0], \\
C^{(1,2)} = [0 \quad 0.07],\quad D^{(0)} = [0.1],\quad D^{(1,1)} = D^{(1,2)} = [0], \\
N_b = 2,\quad f_1(p(t)) = p(t),\ f_2(p(t)) = p(t)^2.
\end{gathered} \tag{12}$$

The system has one input, one output and one scheduling parameter that can take values in the operating range $p \in [3, 7]$. The objective of this example is to see whether the proposed NLS identification method can compete with a state-of-the-art local technique. As a reference, we take the SMILE (*State-space Model Interpolation of Local Estimates*) technique, see de Caigny et al. (2013).

The data used for the identification are five FRFs based on the input-output data sets obtained for fixed scheduling parameter values $p = 3, 4, 5, 6, 7$. Each FRF G_m contains 100 frequency lines, equally distributed between 0 and π. The SMILE technique starts from LTI models identified for fixed values of the scheduling parameter. In the considered case, these LTI models have been identified using the nonlinear least squares frequency domain identification (Pintelon and Schoukens (2012)). In the first step, the SMILE techniques transforms the LTI models to the same state-space basis and, in the end, interpolates the coherent LTI models. For this interpolation, a set of basis functions has to be selected, as well as to parameterize LPV model (1) before the NLS optimization. To make a fair comparison between the two methods, both methods will use the same exact set of basis functions - a second order polynomial.

The initial model parameter guess for the NLS optimization problem (8) is a vector of random variables uniformly distributed in the interval $(0, 0.01)$. Again, we observe the identification in the case when the exact measurements are avaliable, as well as when a zero-mean complex Gaussian noise is added to the local FRFs yielding a signal-to-noise ratio of 60 dB. Each case is repeated 100 times, each time using new realizations of the initial model parameter guess and measurement noise - if added.

The identified LPV models are validated on three different data sets: global validation data, local identification data and local validation data. Global validation data originate from a global experiment where the system is excited with a zero-mean white noise of unit-variance, while the scheduling signal is simultaneously varying as a random phase multisine composed of frequencies in the range $f \in [0.001, 2]$ Hz and with an amplitude covering the operating range. The MSE of the time domain response of the identified model - indicating its overall behavior under changing scheduling parameter conditions with respect to the exact system response - is shown at the top of Fig. 1 for the case without measurement noise, and at the top of Fig. 2 for the case with the measurement noise. Since there are applications where the accuracy of the identified model for particular values of the scheduling parameter is more important than its overall behavior, in the central subplot of the figures one can find the MSE of the frequency response of the identified model with respect to the noise-free local identification FRF data (G_m). Furthermore, 4 extra local FRFs for $p = 3.5$, 4.5, 5.5, 6.5 and without measurement noise are used for validation and the resulting MSE shown at the bottom of Fig. 1 and Fig. 2.

The NLS method outperforms the SMILE technique for all considered data sets except for few where the NLS method converges to a local optimum that corresponds to a worse LPV model. This conclusion holds for both the noise free and the noisy data.

5. EXPERIMENTAL VALIDATION: IDENTIFICATION OF AN OVERHEAD CRANE SYSTEM BY COMBINING LOCAL AND GLOBAL APPROACH

The proposed NLS identification method is validated experimentally on a lab scale model of an overhead crane (see Fig.3). The overhead crane consists of parallel pillars with a traveling bridge in between, along which travels a trolley with a payload attached to it by a cable. The cable length can be varied by a hoisting mechanism in the range of 0.35m to 0.75m. The input to the system is the applied voltage that is proportional to the velocity of the trolley. The system output is the swinging angle with respect to the vertical measured by an encoder. The

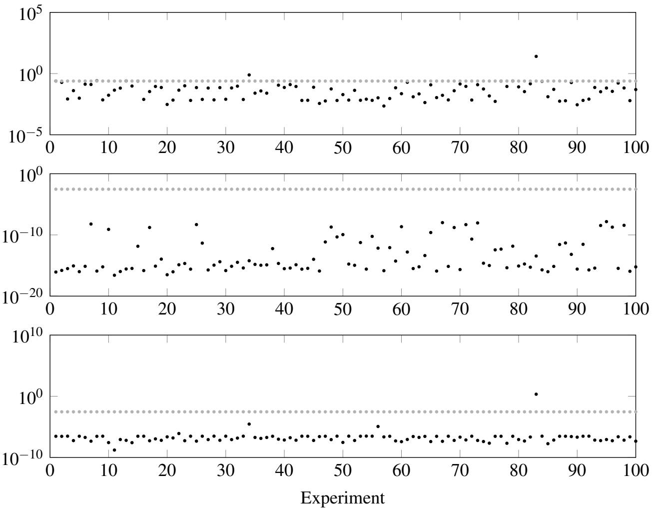

Fig. 1. Validation of the LPV models identified without the measurement noise. Top: MSE of the LPV models identified with the SMILE technique (gray) and with the NLS optimization (black) over 100 proceedings, calculated on time domain validation data from a global experiment. Center: MSE of the FRFs of the identified models calculated with respect to the data used for identification. Bottom: MSE of the FRFs of the identified models calculated with respect to the FRFs taken for validation.

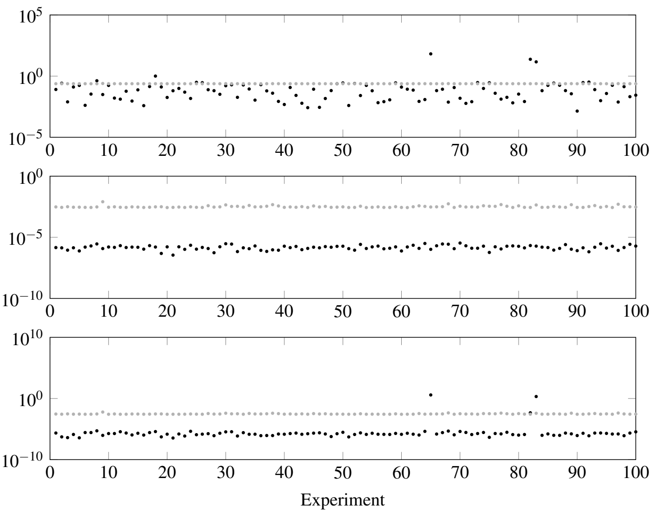

Fig. 2. Validation of the LPV models identified with the measurement noise implying SNR = 60 dB. Top: MSE of the LPV models identified with the SMILE technique (gray) and with the NLS optimization (black) over 100 proceedings, calculated on time domain validation data from a global experiment. Center: MSE of the FRFs of the identified models calculated with respect to the noise-free identification data. Bottom: MSE of the FRFs of the identified models calculated with respect to the FRFs taken for validation.

objective of this example is to identify and validate an LPV model of the described system, with the cable length l as the scheduling parameter. For that purpose, we perform global and local experiments. All experiments are executed at sampling rate $f_s = 100\,\mathrm{Hz}$, and each experiment consists of 6000 time domain data samples of the applied input voltage, measured cable length and measured swinging angle. The excitations are designed such that during the experiments the swinging angle is limited to approximately 14° in order to limit the effect of the system nonlinearities that are proportional to the sine of the angle, and which are not included in the considered LPV model. In all experiments the system is excited with a sequence of two-sided pulse signals, resulting in a sequence of trolley motions from left to right and back, with constant velocity.

During the global experiment the cable length is varying as a random phase multisine signal composed of the frequencies in the range $f \in [0.001, 0.12]$ Hz and taking values in the whole operating range. Two experiments with slightly different settings for the trolley motion and cable length were performed: one for identification and one for validation.

In the local experiments similar sequences of two-sided pulses are applied to the system, while the cable length was fixed. Nine local experiments, with

$$l = 0.35,\ 0.4,\ 0.45,\ 0.5,\ 0.55,\ 0.6,\ 0.65,\ 0.7,\ 0.75\,\mathrm{m}$$

respectively, were performed. The odd experiments are used for identification, and the even for validation.

For the parameterization of the LPV model (1), the following set of basis functions is chosen:

$$f_1 = l(t),\quad f_2 = \frac{1}{l(t)^2},\quad f_3 = \frac{l(t) - l(t-1)}{l(t)}.$$

This choice was inspired by the continuous-time equations of motion of the overhead crane, where similar continuous-time dependencies on the cable length were present. The first two basis functions cover static scheduling dependency, while the third emulates the dependency on the time derivative of the cable length and represents dynamic scheduling dependency.

Global and local identification data are combined in one objective function (5), with the weighting (6) calculated separately for global and local data, and multiplied by weighting scalars α and β, respectively, such that $\alpha + \beta = 1$, $\alpha > 0$, and $\beta > 0$. The introduced weighting scalars give the freedom to balance between the two seemingly exclusive approaches. Two borderline cases are examined: $\alpha = 0.005$ and $\alpha = 0.995$.

As the initial guess for the model parameters (3), the LPV model obtained with the SMILE technique is taken. Based on the local identification data, we first identified five local LTI state-space models using the subspace method `n4sid` from MATLAB System Identification Toolbox, followed by a refinement using `pem`. The SMILE technique transforms the obtained LTI models to a coherent state-space basis and interpolates them with the basis functions f_1 and f_2, since local identification techniques cannot identify dynamic scheduling dependency. The parameters linked with the dynamic basis function f_3 are set to zero. The results obtained with the NLS method and the SMILE technique are summarized in Table 3. The global identification and validation results are shown in Fig.4 and Fig.5, while Fig.6 and Fig.7 respectively cover the local identification and validation for two values of the cable length.

One can see that giving more importance to the global behavior ($\alpha = 0.995$) generally results in a better global fit (Fig. 4 and Fig. 5). Giving more importance to the local behavior ($\alpha = 0.005$) leads to better local fit for the cable lengths that were used in the identification (Fig. 6). The local validation, however, showed no significant difference between the two models.

Table 3. Mean squared error $[°]^2$ obtained with the identified NLS LPV models and the initial LPV model identified with the SMILE technique.

α	GLOBAL IDENTIFICATION DATA	GLOBAL VALIDATION DATA	LOCAL IDENTIFICATION DATA	LOCAL VALIDATION DATA
0.005	$7.7 \cdot 10^{-3}$	$2.86 \cdot 10^{-2}$	$8.1 \cdot 10^{-3}$	$2.39 \cdot 10^{-1}$
0.995	$4.7 \cdot 10^{-3}$	$1.72 \cdot 10^{-2}$	$5.21 \cdot 10^{-2}$	$2.88 \cdot 10^{-1}$
SMILE	$2.75 \cdot 10^{-1}$	$2.67 \cdot 10^{-1}$	$3.08 \cdot 10^{-1}$	$3.74 \cdot 10^{-1}$

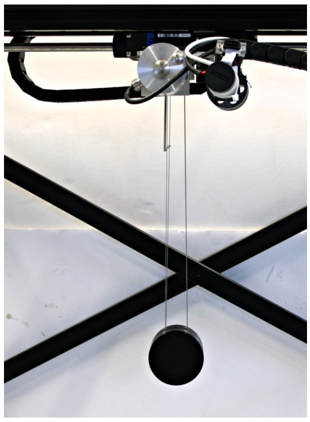

Fig. 3. Lab scale model of an overhead crane

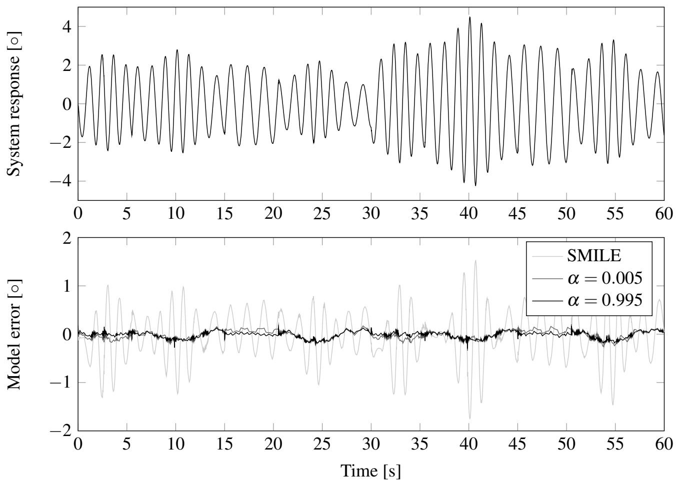

Fig. 4. Global identification results of: the initial model yielding from the SMILE technique, the NLS LPV model with emphasized local behavior ($\alpha = 0.005$), and the NLS LPV model with emphasized global behavior ($\alpha = 0.995$).

Although not surprising, the results confirm the potential of combining global and local data when available, because both NLS models outperform the SMILE technique.

6. CONCLUSION

In this paper we explored possibilities of identifying linear parameter-varying (LPV) systems by combining data coming from global and local experiments into a nonlinear least squares (NLS) optimization criterion. The proposed NLS LPV identification approach proved to be efficient both when used as a global and a local technique, outperforming state-of-the-art

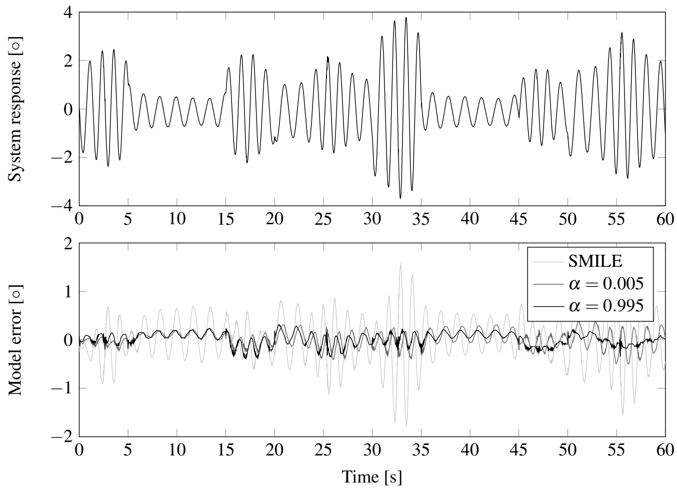

Fig. 5. Global validation results.

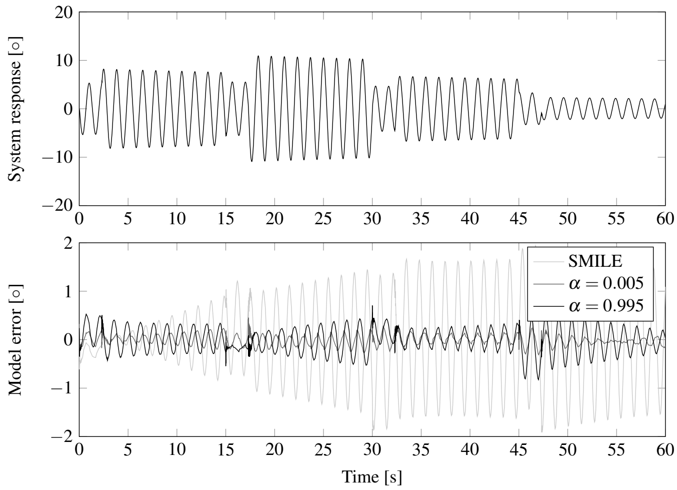

Fig. 6. Local identification results obtained for $l = 0.45\,\mathrm{m}$.

methods, which was shown on two numerical examples. What is particularly notable about the developed NLS approach is that it allows to balance between the local and the global model accuracy, and to find a trade-off between them. This feature was experimentally demonstrated on a lab scale model of an overhead crane. Showing the ease and potential of that solution was the main purpose of this paper.

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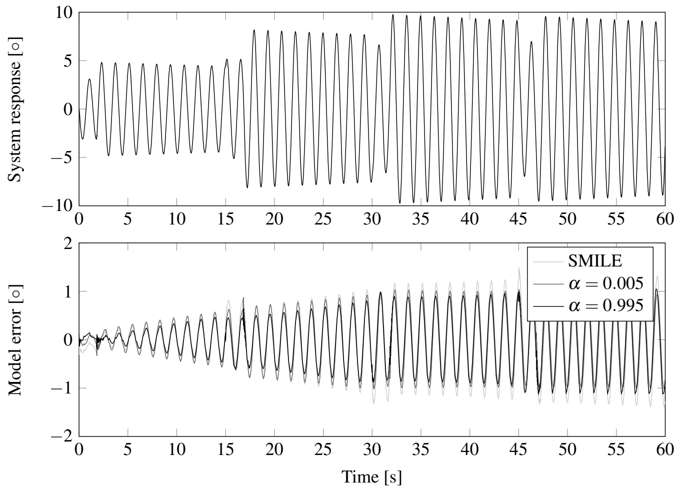

Fig. 7. Local validation results obtained for $l = 0.5\,\mathrm{m}$.

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